First- and Second-Order Matrix Elements for a Curved, Inclined Magnetic Field Boundary. Fringing Field Effects.

Matrix elements for the fringing fields of bending magnets have been derived using an impulse approximation.[7,8] These computations combined with a correction term[9] to the R_{43} element (to correct for the finite extent of actual fringing fields) have produced results which are in substantial agreement with precise ray tracing calculations and with experimental measurements made on actual magnets.

We introduce four new variables (illustrated in Fig. 11); the angle of inclination β_1 of the entrance face of a bending magnet, the radius of curvature R_1 of the entrance face, the angle of inclination β_2 of the exit face, and the radius of curvature R_2 of the exit face. The sign convention of β_1 and β_2 is considered positive for positive focusing in the transverse (y) direction. The sign convention for R_1 and R_2 is positive if the field boundary is convex outward; (a positive R represents a negative sextupole component of strength $k_s^2 L = -\left(\frac{h}{2R}\right) \sec^3\beta$). The sign conventions adopted here are in agreement with Penner,[4] and Brown, Belbeoch, and Bounin.[7]

The results of these calculations yield the following matrix elements for the fringing fields of the entrance face of a bending magnet:

$$R_{11} = 1$$

$$R_{12} = 0$$

$$T_{111} = -\frac{h}{2}\tan^2\beta_1$$

$$T_{133} = \frac{h}{2}\sec^2\beta_1$$

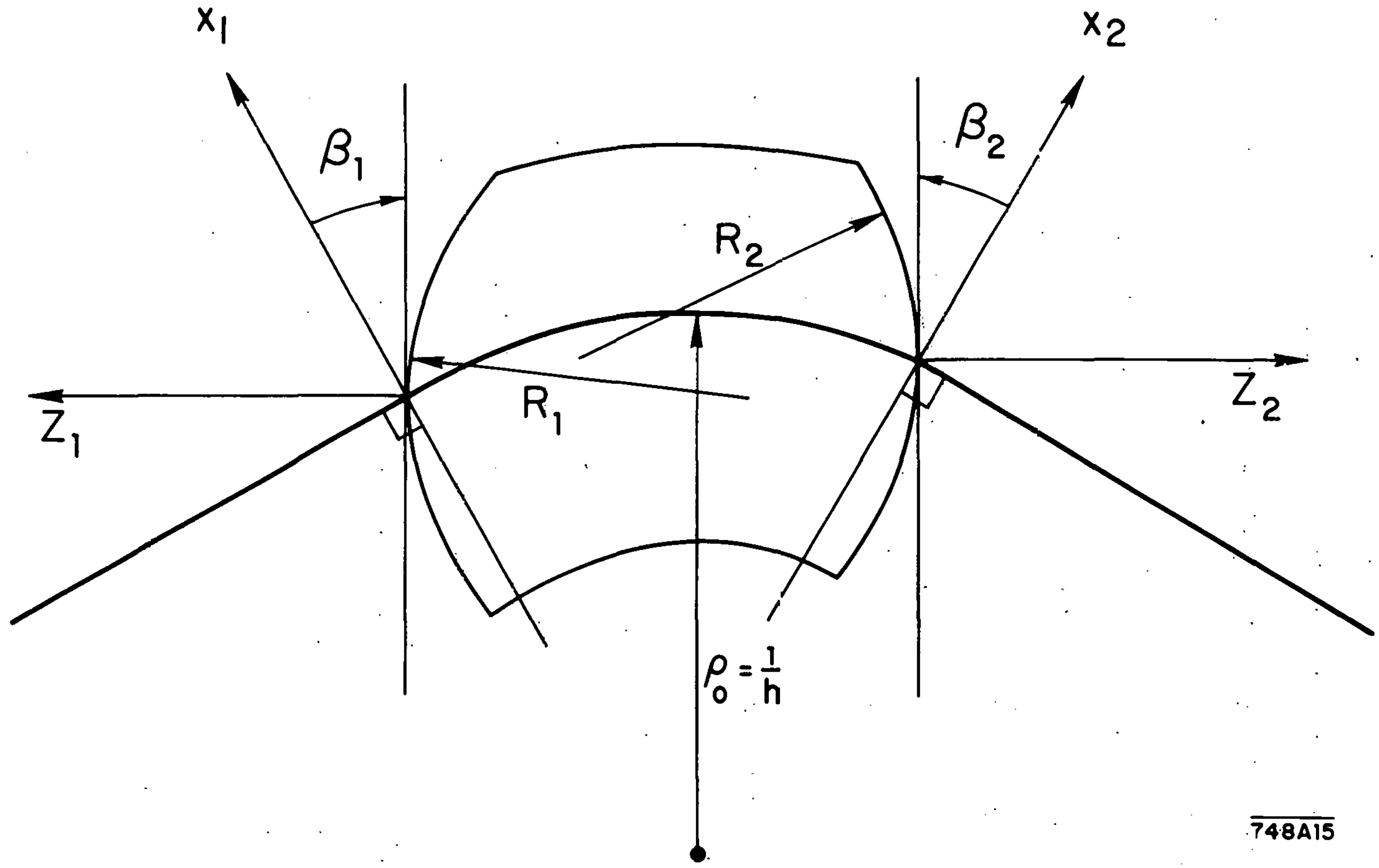

FIG. 11--FIELD BOUNDARIES FOR BENDING MAGNETS

The TRANSPORT sign conventions for x, β, R and h are all positive as shown in the figure. The positive y direction is out of the paper. Positive β's imply transverse focusing. Positive R's (convex curvatures) represent negative sextupole components of strength $S = \left(-\frac{h}{2R}\right)\sec^3\beta$. (See page 71.)

$$R_{21} = -\frac{1}{f_x} = h \tan \beta_1$$

$$R_{22} = 1$$

$$T_{211} = \frac{h}{2R_1} \sec^3 \beta_1 - nh^2 \tan \beta_1$$

$$T_{212} = h \tan^2 \beta_1$$

$$T_{216} = - h \tan \beta_1$$

$$T_{233} = h^2 \left(n + \frac{1}{2} + \tan^2 \beta_1\right) \tan \beta_1 - \frac{h}{2R_1} \sec^3 \beta_1$$

$$T_{234} = - h \tan^2 \beta_1$$

$$R_{33} = 1$$

$$R_{34} = 0$$

$$T_{313} = h \tan^2 \beta_1$$

$$R_{43} = -\frac{1}{f_y} = - h \tan (\beta_1 - \psi_1)$$

$$R_{44} = 1$$

$$T_{413} = -\frac{h}{R_1} \sec^3 \beta_1 + 2h^2 n \tan \beta_1$$

$$T_{414} = - h \tan^2 \beta_1$$

$$T_{423} = - h \sec^2 \beta_1$$

$$T_{436} = h \tan \beta_1 - h\psi_1 \sec^2(\beta_1 - \psi_1) \tag{57}$$

All nonlisted matrix elements are equal to zero. The quantity ψ_1 is the correction to the transverse focal length when the finite extent of the fringing fields are included.[9]

$$\psi_1 = K_1 hg\ (\sec \beta_1)(1 + \sin^2 \beta_1) + \text{higher order terms in } (hg)$$

where g = the distance between the poles of the magnet at the central orbit (i.e., the magnet gap) and

$$K_1 = \int_{-\infty}^{+\infty} \frac{B_y(z)\left[B_o - B_y(z)\right]}{gB_o^2}\, dz$$

$B_y(z)$ is the magnitude of the fringing field on the magnetic mid-plane at a position z. z is the perpendicular distance measured from the entrance face of the magnet to the point in question. See Fig. 11. B_o is the asymptotic value of $B_y(z)$ well inside the magnet entrance. Typical values of K_1 for actual magnets may range from 0.3 to 1.0 depending upon the detailed shape of the magnet profile and the location of the energizing coils.

The matrix elements for the fringing fields of the exit face of a bending magnet are:

$$R_{11} = 1$$

$$R_{12} = 0$$

$$T_{111} = \frac{h}{2} \tan^2 \beta_2$$

$$T_{133} = -\frac{h}{2} \sec^2 \beta_2$$

$$R_{21} = -\frac{1}{f_x} = h \tan \beta_2$$

$$R_{22} = 1$$

$$T_{211} = \frac{h}{2R_2} \sec^3 \beta_2 - h^2\left(n + \frac{1}{2}\tan^2 \beta_2\right) \tan \beta_2$$

$$T_{212} = -h \tan^2 \beta_2$$

$$T_{216} = - h \tan \beta_2$$

$$T_{233} = h^2\left(n - \frac{1}{2} \tan^2 \beta_2\right) \tan \beta_2 - \frac{h}{2R_2} \sec^3 \beta_2$$

$$T_{234} = h \tan^2 \beta_2$$

$$R_{33} = 1$$

$$R_{34} = 0$$

$$T_{313} = - h \tan^2 \beta_2$$

$$R_{43} = - \frac{1}{f_y} = - h \tan(\beta_2 - \psi_2)$$

$$R_{44} = 1$$

$$T_{413} = - \frac{h}{R_2} \sec^3 \beta_2 + h^2(2n + \sec^2 \beta_2) \tan \beta_2$$

$$T_{414} = h \tan^2 \beta_2$$

$$T_{423} = h \sec^2 \beta_2$$

$$T_{436} = h \tan \beta_2 - h\psi_2 \sec^2 (\beta_2 - \psi_2) \tag{58}$$

All nonlisted matrix elements are zero.

$$\psi_2 = K_1 hg \sec \beta_2(1 + \sin^2 \beta_2) + \text{higher order terms in } (hg)$$

and K_1 is evaluated for the exit fringing field.